ASSIGNMENT PROBLEM

-- It is a special type of Linear Programming Problem in which the objective case to find the optimum allocation of a no. of jobs to an equal no. of persons.

The assignment problem can be stated in the form of $n \times n$ matrix $[C_{ij}]$ called cost or effectiveness matrix.

where C_{ij} is the cost of assigning i-th person to j-th job.

Cost or effectiveness Matrix

Jobs

$$\begin{array}{cc|ccccc|}
 & & 1 & 2 \cdots & j \cdots & & n \\
\hline
 & 1 & C_{11} & C_{12} & \cdots\cdots C_{1j} & & C_{1n} \\
 & 2 & C_{21} & C_{22} & \cdots\cdots C_{2j} & \cdots & C_{2n} \\
\text{Person} & \vdots & & & & & \\
 & i & C_{i1} & C_{i2} & \cdots\cdots C_{ij} & \cdots & C_{in} \\
 & \vdots & & & & & \\
 & n & C_{n1} & C_{n2} & \cdots\cdots C_{nj} & \cdots & C_{nn} \\
\hline
\end{array}$$

Mathematical Formulation of Assignment Problem

— Mathematically an assignment problem can be stated as follows —

Minimize the total cost

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

where, $x_{ij} = 1$ if i-th job is assigned to j-th person

$= 0$ if i-th job is not assigned to j-th person.

subject to the conditions

i) $\sum_{i=1}^{n} x_{ij} = 1$, $j = 1, 2, \ldots n$

which means that only one job is done by the i-th person.

2) $\sum_{j=1}^{n} x_{ij} = 1$, $i = 1, 2, \ldots n$

which means that only one person should be assigned the j-th job.

HUNGARIAN METHOD FOR SOLVING AN ASSIGNMENT PROBLEM

Step I:
Subtract the minimum element of each row in the cost matrix $[C_{ij}]$ from every element of the corresponding ~~group~~. row.

Step II:
Subtract the minimum element of each column in the reduced matrix obtained in step-I from every element of corresponding Column.

Step III:

a) Starting with row 1 of the matrix obtained in step II, examine rows successively until a row with exactly one zero element is found. Mark □ at this Zero as an assignment will be made there. Mark X at all other Zeros in the Column (in which we mark □) to show that they can not be used to make other assignments.
Proceed in this way until the last row is examined.

b) After examining all the rows completely proceed similarly examining the columns.

c) Continue the operations (a) and (b) successively until we reach to any of the two situations —

i) all the zeros marked ☐ or crossed

OR

ii) the remaining unmarked zeros lie at least two in each row and column.

In case (i) we have a maximum assignment and in case (ii) we have still some zeros to be treated for which we use trial and error method.

Now there are two possibilities —

1) if it has an assignment in every row and every column (i.e. total no. of mark ☐ zeros is exactly n)
Then the complete optimal assignment is obtained.

2) if it does not contain assignment in every row and every column (ie the total no. of mark ☐ zeros is less than n)
Then we have to modify the cost matrix by adding or subtracting to create some more zeros in it.

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for this we proceed to next step.

Step-IV :
when the matrix obtained in step III does not contain assignment in every row and every column then we draw the minimum no. of horizontal and vertical lines, necessary to cover all zeros at least once. for this the following procedure is adopted ——

1) Mark [✓] all rows in which assignment have not been made.

2) Mark [✓] in the columns which have crossed zeros in marked rows

3) Mark [✓] in rows not already marked which have assignment in marked columns.

4) Repeat step (2) and (3) until the chain of marking ends.

5) Draw minimum no. of lines through unmarked rows and marked colums to cover all the zeros

Step V

— Select the smallest of the elements that do not have any line through them, subtract it from all the elements that do not have a line through them. Add it to every element that lies at the intersection of two lines and leave the remaining elements of the matrix unchanged.

Step VI :

At the end of step V, no of zeros are increased in the matrix.
Now we apply step III in the modified matrix obtained in step V to obtain the desired solution.